



MS3
£3.00

GCSE MARKING SCHEME

MATHEMATICS (TWO TIER)

SUMMER 2009

INTRODUCTION

The marking schemes which follow were those used by WJEC for the Summer 2009 examination in GCSE MATHEMATICS (TWO TIER). They were finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conferences were held shortly after the papers were taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conferences was to ensure that the marking schemes were interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conferences, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about these marking schemes.

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Paper 1 - Higher Tier

Paper 1 2009 Higher Tier		Comments												
1.(a) 70 (kg), 175 (cm) (b) Positive (c) Suitable line, with some points above and below (d) Answers in the range >50 (kg) but ≤65 (kg) (e) No, not plotted in order OR similar explanation	B2 B1 B1 B1 B1 B1 6	B1 for either or if reversed Do not accept a description No requirement to pass through the means OR Suitable answer from their line of best fit No maybe implied in their statement												
2. (a) Entries of 1000 (ml) and 1000 (ml) 16/4 x 115 or other complete method <div style="text-align: right; margin-right: 100px;">460(g) 60 (ml)</div> (b) 1000ml = 1 litre <div style="text-align: right; margin-right: 100px;">(5500x8) / their (a) entry for milk 44 (people)</div>	B1 M1 A1 A1 B1 B1 M1 A1 A1 7	Maybe implied by use of 1000ml & 5500ml OR 1litre & 5.5litres, or from their milk entry OR (5.5x8) / their (a) entry for milk for M1 only FT their entry for milk in (a) SC1 for 44 with incorrect place value												
3. (a) Sight of any two from: 50, 400 and 200 <div style="text-align: right; margin-right: 100px;">100</div> (b) 4280 (c) 90 (%) (d) Attempt common format of 3 given values 13/20, 8/20, 6/20 with 5/20 or equivalent 3/10 or its equivalent	M1 A1 B1 B1 M1 A1 A1 B1 7	Maybe implied if correct response is given. Accept values that could lead to a simple calculation For their simple calculation, e.g 98 All decimals, common denominator or % Or 65%, 40%, 30% with 25%, or as decimals												
4. (a)(i) Sight of 10 or +6 within the substitution <div style="text-align: center; margin-left: 100px;">8</div> (ii) Sight of $3 \times 5 \times 4$ or $3 \times 5 \times -2^2$ <div style="text-align: center; margin-left: 100px;">60</div> (b) $6n + 5$ or equivalent ISW (c) (i) $7n - 2$ (ii) $20^2 - 2$ OR $20 \times 20 - 2$ <div style="text-align: right; margin-right: 100px;">= 398</div>	B1 B1 B1 B1 B1 B1 B2 M1 A1 9	$\{10 - (-6)\}/2$ CAO If no B marks award SC1 for -60 Accept $6 \times n + 5$ B1 for $7 \times n$ seen, (not $n+7$), or "term in n " -2 MR-1 for finding 12 th term (No marks for 38)												
5.(a) <table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <tr><td style="width: 33%;">/</td><td style="width: 33%;"></td><td style="width: 33%;"></td></tr> <tr><td>/</td><td>/</td><td></td></tr> <tr><td></td><td></td><td>/</td></tr> <tr><td>/</td><td></td><td></td></tr> </table> (b) $3 + 5 < 10$ or equivalent in words (c) Sight of 30 and 70 OR exterior total is 360 $180 - (30+70)$ and intention to subtract this from 180 OR $360 - (150+110)$ <div style="text-align: right; margin-right: 100px;">$100^{(0)}$</div>	/			/	/				/	/			B3 B1 M1 M1 A1 7	B2 for any 2 columns correct or any 2 rows correct B1 for any 1 column correct or any 1 row correct For understanding of triangle formation. Ignore 10 stated as hypotenuse Intention rather than accurate notation CAO. SC1 for 80 if no other marks awarded
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6. (a) $1.5\text{kg} = 1500 \text{ (g)}$ $1500/300$ or $1.5/300$ <div style="text-align: right; margin-right: 100px;">$5 \text{ (g/cm}^3\text{)}$</div> (b) (10 minutes =) 600 (seconds) $600/50$ (=12 buckets) 48 (litres) OR equivalent (c) 158 (mm)	B1 M1 A1 B1 M1 A1 B1 7	Stated or implied by x1000 CAO Seen or implied in working, or reverse 50/60 hrs Or alternative method CAO. When units are given they should be correct Accept 157.9												
7.(a) $c^4 - 5c$ (b) $8d - 12 + 15 - 21d$ <div style="text-align: center; margin-left: 100px;">= - 13d + 3</div> (c)(i) $7e + 12 = 3e + 18$ $7e - 3e = 18 - 12$ OR $4e = 6$ <div style="text-align: center; margin-left: 100px;">$e = 6/4$ (= 1.5)</div> (ii) $f/5 = 17 - 7$ OR $f/5 = 10$ OR $f + 35 = 17 \times 5$ <div style="text-align: center; margin-left: 100px;">$f = 50$</div>	B1 B1 B1 B1 B1 B1 B1 B1 B1 8	Not ISW FT their expand if not more than 1 error Expand bracket. FT until 2 nd error in (c) Collection like terms, no need to see simplified ISW CAO												

Paper 1 2009 Higher Tier		Comments
8. (a) 0.6, 0.3, 0.7, 0.3, 0.7 (b) 0.4×0.7 $= 0.28$	B2 M1 A1 4	B1 0.6 with one other correct, or 0.3 & 0.7 once FT their 0.7
9.(a)(i) 4.7×10^{-3} (ii) 3.2×10^4 (b) 6.3×10^3 (c) $10/3 \times 6/5$ $= 4$ (d) 19 (e) 1/81	B1 B1 B2 B1 B1 B1 B1 8	Penalise incorrect notation on the 1st occasion only B1 for 6300 or $\dots \times 10^3$ CAO. Do not accept an improper fraction.
10. Sight of terms $5x$ and $5x + 12$ Their expression of 3 terms = 672 $x + 5x + 5x + 12 = 672$ $x = 60$ (litres of black paint) 300 and 312 (litres)	B1 M1 A1 B1 B1 5	Accept “ \times ” sign included Correct equation CAO FT their x for $5x$ and $5x+12$, provided at least 1 mark already awarded <i>SC2 for 60, 300 and 312 only, no equation, OR SC1 for 60 litres of black paint, no equation</i>
11.(a) -19 (b) Suitable scale Plotting 4 points correctly Smooth curve through 5 correct points (c) Straight line $y = x + 4$ seen Approximately (1.7 , 5.5), use their graph	B1 M1 M1 A1 B1 B1 6	Not off the paper provided FT from (a) FT from (a) CAO. Tolerance of to a square on each axis
12. (a) (i) $(5x + 7y)(5x - 7y)$ (ii) Denominator factorised $2x(5x - 7y)$ $(5x+7y)/2x$ (b) $(5x + 3)(3x + 2)$ $-3/5$ and $-2/3$ ISW	B2 B1 B1 B2 B1 7	B1 for $(5x - 7y)(5x - 7y)$ or one slip FT from (i) if equivalent problem Penalise further working -1 B1 for $5x, 3x, 3$ and 2 FT their pair of brackets
13. (a) Entries 30, 30, 40, 35, 5 (b) Attempt to find $8/10$ 30 to 40 group ($= 8/10 \times 40$) $= 32$ 72 (cars) (c) 3, 4, 2, 1.5, 0.5 Axes correct and labelled, no gaps between bars Correct histogram (d) Yes, with reason (e.g. more slower)	B2 M1 M1 A1 B1 M1 A1 B1 9	B1 for any 3 correct entries FT for their entries for M marks only in (b) OR $2/10$ of 30 to 40 group ($2/10 \times 40$) OR $140 - (8 + 30 + 30)$ CAO Histogram needs to be attempted FT candidates frequency density if table completed incorrectly but the idea of frequency density is used. SC1 if correct but not labelled. FT from their histogram in (b) if necessary
14. $y=2+\cos x$, $y=\tan x$, $y=\sin x$	B2 2	B1 for any one correct
15. AB = b - a OR AC = 2/5AB OR AC = 2/5(b-a) OR CB = 3/5AB OR CB = 3/5(b - a) OR OC = a + 2/5 AB OC = a + 2/5(b - a) OR OC = b - 3/5(b - a) OC = 3/5a + 2/5b	M1 A1 A1 3	OR equivalent OR equivalent CAO
16. (a) All 5 points plotted correctly (b) $b \approx 100$ Gradient = difference y / difference x^2 $a \approx 8$	B2 B1 M1 A1 5	B1 for one error Or alternative method to find a ≈ 10

2009 Summer Paper 2 (Calculator allowed) Foundation Tier	Marks	FINAL POST COFERENCE MARK SCHEME Comments (06/06/2009) (Page 2)
8. (a =) 3 (b =) 6 (c =) 7 (d =) 5	B1 B1 B1 B1 4	C.A.O. F.T 9 – a F.T. 22 – 2b – a F.T. 21 – (a +b+c)
9. (a) (i) 29 (ii) $(17 + 3)/4 = 5$ (b) (i) add three (to the previous term) (ii) halve or divide by two	B1 M1 A1 B1 B1 5	C.A.O. <u>Accept embedded answers such as $5 \times 4 - 3 = 17$</u> Accept +3 Accept $\div 2$
10. A (12, 4) B (0, 8) C (16, 16)	B1, B1 B1, B1 B1, B1 6	C.A.O. <u>REVERSED COORDINATES GET NO MARKS.</u> C.A.O. C.A.O. <u>Penalise –1 once only for incorrect notation such as (12x, 4) OR (x=12, y=4)</u>
11. (a) 4 –6 (b) (x =) 32 (c) – 13	B1 B1 B1 B2 5	C.A.O. F.T. ‘their 4’ – 10 if negative <u>Accept embedded answers such as $32/4 = 8$</u> B1 for the –3 OR – 10 <u>–3a – 10b gets B0.</u>
12. (a) 21/50 <u>ISW</u> (b) $(0 \times 3) + 1 \times 12 + 2 \times 14 + 3 \times 18 + 4 \times 2 + 5 \times 1 = 107$	B2 M1 A1 4	<u>B1 for a numerator of 21 OR B1 for a denominator of 50 if in a fraction < 1</u> <u>Accept percentages 42% OR decimals .42</u> <u>Adding two fx terms is enough, e.g. $1 \times 12 + 2 \times 14$</u> C.A.O.
13. (a) $\frac{36}{100} \times 72$ $= 25.92$ <u>ISW</u> (b) $\frac{78}{120} \times 100$ $= 65 (\%)$	M1 A1 M1 A1 4	<u>Accept other methods such as $10\%=7.2, 30\%=21.6, 5\%=3.6, 1\%=.72$ as long as the partitioning would lead to 25.92 if calculated correctly</u> C.A.O. <u>Ignore % sign in the answer.</u> <u>C.A.O.</u>
14. 3 or 4 angles correct and correctly labelled. 3 or 4 angles correct, labels not fully correct. 2 angles correct and correctly labelled. 2 angles correct, labels not fully correct. 1 angle correct and correctly labelled. OR <u>If 0 OR 1 for their diagram or no diagram,</u> 360/120 Angles are 126, 81, 105 and 48	B4 OR B3 B3 B2 B1 M1 A1 4	Use the overlay and allow $\pm 2^\circ$. Correct labels (Words NOT the frequency OR angle). 3 correct labels is enough. If only B1 is scored for the diagram, and all the angles given correctly, then cancel the B1 and award M1, A1 for 2 marks. If B0 scored for the diagram, check the angles and the method to see if the M1 and the A1 can be awarded. (1 is) 3° gets the M1. OR SC1 for all correct percentages: 35, 22.5, 29.2, 13.3 <u>OR 35, 22 or 23, 29, 13</u>
15. (a) Distance = <u>10.2 – 10.6</u> (cm) $= \div 2$ $=$ <u>5.1 – 5.3</u> (km) (b) Bearing Distance	B1 M1 A1 B1 B1 5	Allow $10.4 \pm 2\text{mm}$ F.T. their distance Use overlay <u>Use overlay</u>

Paper 2 - Higher Tier

Paper 2 2009 Higher Tier		Comments
1.(a) Strategy, knowing that the probabilities add to 1 0.13 (b) Yellow and Blue (or 0.26 and 0.37) (c) $0.24 + 0.37 = 0.61$	M1 A1 B1 M1 A1 5	E.g. Attempt to add all and subtract from 1, or noticing first 2 make 0.5 & working towards 0.5 FT their (a) if greater than either of these
2. (a) Enlargement scale factor 2 Correct position (b) Correct reflection in $x = 1$ (c) Correct translation (d) Bottom right shape indicated	B2 B1 B2 B1 B1 7	B1 3 lines correct, or consistent incorrect scale FT consistent incorrect scale B1 a reflection in $y=1$ or either axis, OR for drawing $x=1$. MR-1 for use of $x=-1$ Accept any unambiguous indication
3.(a) 57 / 1h 30 mins $57/1.5$ or equivalent 38 (mph) (b) $(12/100) \times 132 + 132$ OR $1.12 \times 132 = (\pounds) 147.84$ (c) $400 \times 1.44 = 576$ (Euros) $(576 - 180 =) 396$ (Euros left) their 396 / 1.44 = $(\pounds)275$ (d) 8.1	M1 A1 A1 M2 A1 M1 A1 B1 M1 A1 B2 13	Method only, e.g. implied by 43.8..., 0.63..., 0.438 CAO M1 for $(12/100) \times 132 (= 15.84)$, or 116.16 ISW OR $180/1.44$ M1 $= 125$ (pounds spent) A1 $400 - \text{their } 125$ M1 $= (\pounds) 275$ A2 CAO CAO <i>SCI for an answer 152.77or 152.78</i> B1 for 8 or 8.05263.....
4. Strategy of finding prime factors $24 = 2 \times 2 \times 2 \times 3$ and $54 = 2 \times 3 \times 3 \times 3$ $24 \times 54 = 2^4 \times 3^4$ or $2^2 \times 2^2 \times 3^2 \times 3^2$ or $2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3$ Explanation, even powers or occurs in pairs <i>(Accept perfect square from correct prime factor working)</i>	M1 M1 A1 E1 4	At least 2 prime factors for either value For prime factors, not notation Or implied by the explanation of prime factors and even indices <i>SCI for $2^2 \times \dots$ or $3^4 \times \dots$</i> FT from $3^4 \times 4^2$ or $2^4 \times 9^2$.
5. Mid-points 32, 36, 40, 44, 48 $\frac{32 \times 2 + 36 \times 9 + 40 \times 30 + 44 \times 11 + 48 \times 8}{60}$ (OR 2456) $= 40.9$ (..(or 40.93 or 40.93 OR 41 (seconds)	B1 M1 M1 A1 4	Two shown is sufficient if no error Attempt $\sum fx$ for their mid-points Attempt their $\sum fx$ divided by 60. CAO
6. 1.4 gives 19.(04) , 1.5 gives 22.875 (22, 22.8, 22.87, 22.9, 23) Alternative: -20 then compare with 0 Explanation that one calculated value is too small and the other calculated value is too large, these calculated values need to be shown and used. Use of inequality signs with calculated values without words is acceptable. <i>Accept arrows as indication of too small, too large – both are required</i>	B1 E1 2	Or two other values , giving <20 , the other >20 . $1.41 \quad 19.402305 \quad -0.597695$ $1.42 \quad 19.76924$ $1.43 \quad 20.140835$ $1.44 \quad 20.51712$ $1.45 \quad 20.898125$ $1.46 \quad 21.28388$ $1.47 \quad 21.674415$ $1.48 \quad 22.06976$ $1.49 \quad 22.469945 \quad 2.469945$
7. $AB^2 = 5.3^2 - 2.8^2$ OR sine rule, with values $= 20.25$ OR correct rearranged sine rule $AB = 4.5(\text{cm})$ OR Angle A $(31.89 \dots^{\circ})$ Area cross section $= \frac{1}{2} \times 2.8 \times AB$ OR other correct method OR embedded in volume calculation Volume = area cross section $\times 3.5$ $22.(05 \text{ cm}^3)$ or $22.1 (\text{cm}^3)$	M1 A1 A1 M1 M1 A1 6	OR $\cos C = 2.8/5.3$ M1 $C = 58(1 \dots^{\circ})$ A2 (A1 for 0.52..) $\frac{1}{2} \times 5.3 \times 2.8 \times \sin C$ M1 (6.3) \downarrow FT their area x-section CAO
8.(a) $q + 45 = 8p$ OR $-8p = -q - 45$ $p = (q + 45) / 8$ or equivalent ISW (b) $15t - 4t < 7$ $t < 7/11$ ISW <i>No marks for use of "=" unless replaced at the final stage, then allow all possible marks</i>	B1 B1 B1 B1 4	FT $q - 45 = 8p$ only to $p = (q - 45) / 8$ Accept $t < 0.6$ from correct working. FT if only one error in first stage.

Paper 2 2009 Higher Tier	Comments	
9.(a) 15, 50, 60 (b) Idea, plotting upper class boundary (consistent)with the corresponding cumul. freq. 2 points plotted correctly All points correct and joined by straight lines or curve (c) Median First group by	B1 M1 A1 A1 B1 B1 6	<i>In (b) accept plots at 20,40,60,80 OR 19.5,39.5,59.5,79.5</i> FT their cumulative table of values for all marks, must be cumulative <i>SC1 if plotted and joined using mid-points or -1 (approximately 49)</i> FT their cumulative diagram in(c) FT logic, group with difference from 40. The median maybe implied by the difference
10.(a) 4500 / 15 (= 300) (£)600, (£) 1500, (£) 2400 (b) Area = $\frac{1}{2} \Pi 28^2$ OR $\frac{1}{2} \Pi 2.8^2$ 1230 to 1232 (mm ²) 12.30 to 12.32 (cm ²) (c) $68.25 / 105 \times 100$ = 65 3.25 (dollars)	M1 A2 M1 A1 M1 A1 A1 8	A1 for any one answer Ignore final incorrect conversion of units <i>SC1 for 24(.)60 to 24(.)6 or 2460 to 2464</i> Maybe implied within final A1 Ignore £ given <i>SC1 for sight of 1.05 or 105 if no other marks</i>
11.(a) $6x^9y^9$ (b) $x^2 + 8x - 2x - 16$ = $x^2 + 6x - 16$ (c) $a^2(a + 4)$	B2 B1 B1 B1 5	B1 for an two of the three correct, e.g. $6x^9y^9$, i.e. -1 per error Any three of the 4 terms correct CAO. Penalise further working -1
12. Overall strategy BC = 19.6 x tan 27 BC = 9.986.... AC = 17.5 + BC Tan ADC = AC/19.6 Tan ADC = 1.4(023...) <ADC = 54 ⁰ to 55 ⁰	B1 M2 A1 B1 M1 A1 A1 8	M1 for tan27 = BC/19.6 FT their BC (AC=27.486...) FT their AC which is > BC for M1 and A1 mark CAO
General principles for other methods in Q12. B1 overall strategy, M2 A1 for the first relevant application of trigonometry, B1 for simple calculation of a length or angle for the next step, M1 A2 for the last relevant step using trigonometry. E.g Initially use of sine rule with values M1, Rearrange for BC M1, BC = 21.997...,		
13. $3/10 \times 7/9$ or equivalent 2 x Or listing 2 ways only 42/90 (= 7/15) or equivalent	M1 M1 A1 3	$3/10 \times 5/9 + 3/10 \times 2/9$ (would give 21/90) $P(B) \times P(B') + P(B') \times P(B)$ only or alternative complete idea, irrespective of replacement ISW <i>Alternative method: $1 - P(bb) - P(no blue)$ M1</i> <i>Use of correct values M1</i> <i>42/90 A1</i>
14.(a) 56 ⁰ (b) 57 ⁰ (c) 114 ⁰	B1 B1 B1 3	FT 2 (b). Accept 246 ⁰
15.(a) $\sin BAC / 6.6 = \sin 62 / 8.4$ $\sin BAC = \sin 62 / 8.4 \times 6.6$ BAC = 43.9(....) or 44 ⁰ (b) Strategy to find missing angle, 180 – 62 – BAC Area = $\frac{1}{2} 8.4 \times 6.6 \times \sin C$ = 26.6(555....cm ²) or 27 (cm ²)	M1 M1 A1 M1 M1 A1 6	CAO (74) FT their C from their BAC FT their C from their BAC Alternative for (b) AB=9(1..) M1 $\frac{1}{2} 6.6 \times 9.1 \times \sin 62$ M1 26.6(555....cm ²) M1 or 27 (cm ²) A1

Paper 2 2009 Higher Tier		Comments
16.(a) $\frac{1}{3} \Pi x^2 3$ OR $\frac{1}{3} \Pi (x+1)^2 6$ $\frac{1}{3} \Pi x^2 3 + \frac{1}{3} \Pi (x+1)^2 6 = 102 \Pi$ $x^2 + 2(x+1)^2 = 102$ $x^2 + 2x^2 + 4x + 2 = 102$ leading to $3x^2 + 4x - 100 = 0$ (b) $x = \frac{-4 \pm \sqrt{4^2 - 4 \cdot 3 \cdot (-100)}}{2 \cdot 3}$ $= \frac{-4 \pm \sqrt{1216}}{6}$ $x = 5.145 \dots$ Radius larger cone = 6.1 or 6 (cm)	B1 M1 A1 A1 M1 A1 A1 A1 A1 8	Their expression for the sum of volumes, with at least one expression correct, equated to 102Π Eliminate fraction and Π CAO Allow one slip for M mark only Ignore negative solution FT their $x+1$ to whole or 1dp if M1 awarded <i>Trial & Improvement:</i> <i>M1 correct evaluation either side, M1 check to 2dp, A1 for 5.1(...), A1 for 6.1 or 6</i>
17. Tangent used or shown at 12.30 Use of difference distance / difference time $\dots \pm 2 \text{ km/h}$	B1 M2 A1 4	Award M1 only if time unit is minutes <i>No marks for reading at 12 30 for distance then doubling</i>
18. Numerator $x(8x - 1) - 2(3x + 1)$ Denominator $(3x + 1)(8x - 1)$ $8x^2 - 7x - 2$ seen in working $\frac{8x^2 - 7x - 2}{(3x + 1)(8x - 1)}$ or $\frac{8x^2 - 7x - 2}{24x^2 + 5x - 1}$	M1 M1 B1 A1 4	Penalise further working, apart from incorrect expansion of denominator



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